



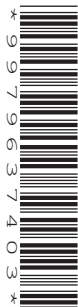
Oxford Cambridge and RSA

Friday 23 June 2023 – Afternoon

A Level Further Mathematics A

Y545/01 Additional Pure Mathematics

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

- Read each question carefully before you start your answer.

- 1 The surface S is defined for all real x and y by the equation $z = x^2 + 2xy$. The intersection of S with the plane Π gives a section of the surface. On the axes provided in the Printed Answer Booklet, sketch this section when the equation of Π is each of the following.

(a) $x = 1$ [2]

(b) $y = 1$ [2]

- 2 A curve has equation $y = \sqrt{1+x^2}$, for $0 \leq x \leq 1$, where both the x - and y -units are in cm. The area of the surface generated when this curve is rotated fully about the x -axis is $A \text{ cm}^2$.

(a) Show that $A = 2\pi \int_0^1 \sqrt{1+kx^2} \, dx$ for some integer k to be determined. [4]

A small component for a car is produced in the shape of this surface. The curved surface area of the component must be 8 cm^2 , accurate to within one percent. The engineering process produces such components with a curved surface area accurate to within one half of one percent.

(b) Determine whether all components produced will be suitable for use in the car. [2]

- 3 The points A and B have position vectors $\mathbf{a} = \mathbf{i} + p\mathbf{j} + q\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ respectively, relative to the origin O .

(a) Determine the value of p and the value of q for which $\mathbf{a} \times \mathbf{b} = 2\mathbf{i} + 6\mathbf{j} - 11\mathbf{k}$. [3]

(b) The point C has coordinates (d, e, f) and the tetrahedron $OABC$ has volume 7.

(i) Using the values of p and q found in part (a), find the possible relationships between d , e and f . [2]

(ii) Explain the geometrical significance of these relationships. [2]

- 4 The sequence $\{A_n\}$ is given for all integers $n \geq 0$ by $A_n = \frac{I_{n+2}}{I_n}$, where $I_n = \int_0^{\frac{1}{2}\pi} \cos^n x \, dx$.

- Show that $\{A_n\}$ increases monotonically.
- Show that $\{A_n\}$ converges to a limit, A , whose exact value should be stated. [7]

- 5 (a) The group G consists of the set $S = \{1, 9, 17, 25\}$ under $\times_{32}$, the operation of multiplication modulo 32.
- (i) Complete the Cayley table for G given in the Printed Answer Booklet. [2]
- (ii) Up to isomorphisms, there are only two groups of order 4.
- C_4 , the cyclic group of order 4
 - K_4 , the non-cyclic (Klein) group of order 4
- State, with justification, to which of these two groups G is isomorphic. [2]
- (b) (i) List the odd quadratic residues modulo 32. [2]
- (ii) Given that n is an odd integer, prove that $n^6 + 3n^4 + 7n^2 \equiv 11 \pmod{32}$. [4]
- 6 The surface S has equation $z = x \sin y + \frac{y}{x}$ for $x > 0$ and $0 < y < \pi$.
- (a) Determine, as a function of x and y , the determinant of $\mathbf{H}$, the Hessian matrix of S . [6]
- (b) Given that S has just one stationary point, P , use the answer to part (a) to deduce the nature of P . [2]
- (c) The coordinates of P are (α, β, γ) .
- Show that β satisfies the equation $\beta + \tan \beta = 0$. [3]
- 7 Binet's formula for the n th Fibonacci number is given by $F_n = \frac{1}{\sqrt{5}}(\alpha^n - \beta^n)$ for $n \geq 0$, where α and β (with $\alpha > 0 > \beta$) are the roots of $x^2 - x - 1 = 0$.
- (a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [1]
- (b) Consider the sequence $\{S_n\}$, where $S_n = \alpha^n + \beta^n$ for $n \geq 0$.
- (i) Determine the values of S_2 and S_3 . [3]
- (ii) Show that $S_{n+2} = S_{n+1} + S_n$ for $n \geq 0$. [2]
- (iii) Deduce that S_n is an integer for all $n \geq 0$. [1]
- (c) A student models the terms of the sequence $\{S_n\}$ using the formula $T_n = \alpha^n$.
- (i) Explain why this formula is unsuitable for every $n \geq 1$. [1]
- (ii) Considering the cases n even and n odd separately, state a modification of the formula $T_n = \alpha^n$, other than $T_n = \alpha^n + \beta^n$, such that $T_n = S_n$ for all $n \geq 1$. [2]

- 8 Let $f(n)$ denote the base- n number 2121_n where $n \geq 3$.
- (a) (i) For each $n \geq 3$, show that $f(n)$ can be written as the product of two positive integers greater than 1, $a(n)$ and $b(n)$, each of which is a function of n . [2]
- (ii) Deduce that $f(n)$ is always composite. [1]
- (b) Let h be the highest common factor of $a(n)$ and $b(n)$.
- (i) Prove that h is either 1 or 5. [4]
- (ii) Find a value of n for which $h = 5$. [2]
- 9 The set C consists of the set of all complex numbers excluding 1 and -1 . The operation $\oplus$ is defined on the elements of C by $a \oplus b = \frac{a+b}{ab+1}$ where $a, b \in C$.
- (a) Determine the identity element of C under $\oplus$. [2]
- (b) For each element x in C show that it has an inverse element in C . [2]
- (c) Show that $\oplus$ is associative on C . [3]
- (d) Explain why $(C, \oplus)$ is not a group. [1]
- (e) Find a subset, D , of C such that $(D, \oplus)$ is a group of order 3. [3]

END OF QUESTION PAPER

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